Bayesian Sensitivity Analysis for Set-identified Structural Models

Yizhou Kuang[†]

[†]Department of Economics, University of Manchester

Last updated: May 8th, 2024

Motivation

- **Set-identified structural models are ubiquitous.**
  - ∗ E.g., DSGE models are widely used:
    - ∗∗ U.S. Fed, Bank of Canada, Sveriges Riksbank, IMF etc.
    - ∗∗ They are also super relevant for policy-making.
- **Analysis of these models is challenging because of 'identification':**
  - ∗ DSGE models are micro-founded, rich with parameters.
  - ∗ Multiple parameter combinations may yield same data generating process.
  - ∗ Standard Bayesian methods can be sensitive to prior choices.

Motivation - Estimation

A monetary policy model (Cochrane 2011, JPE). In its AR(1) form

$$\pi_t = \rho \pi_{t-1} + \frac{1}{\phi_\pi - \rho}\epsilon_t, \quad \phi_\pi > 1, |\rho| < 1, \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

parameter vector $(\phi_\pi, \sigma_\epsilon, \rho)$, Taylor rule parameter ϕ_π, monetary policy disturbance coefficient ρ, its standard error σ_ϵ. Inflation rate π_t is observed.

Motivation - Estimation

Table: Prior and Posterior Distribution of Structural Parameters

	True value	Prior distribution			Posterior distribution			
		Distr.	Mean	St. Dev.	Mode	Mean	5 percent	95 percent
σ_ϵ	1	Uniform	4	2.02	5.82	4.43	1.94	2.02
ϕ_π	1.8	Uniform	4	1.73	6.49	4.91	2.78	7.00
ρ	0.8	Uniform	0.75	0.09	0.82	0.81	0.74	0.87

Likelihood

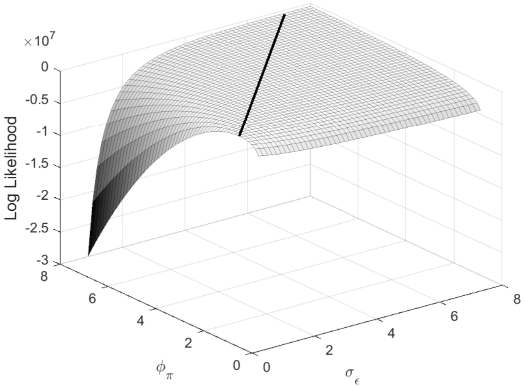
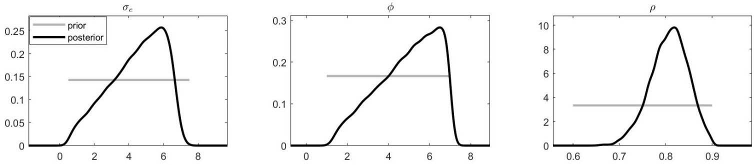


Figure: Likelihood function while fix $\rho = 0.8$, $T = 1,000,000$

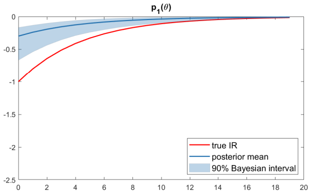
- Maxima along the $\sigma_\epsilon = \phi_\pi - 0.8$ line

Prior Sensitivity

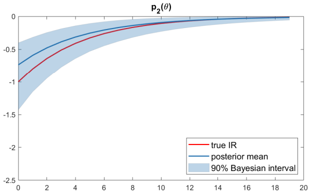


- The posterior of σ_ϵ and ϕ_π are extremely informative even if only $\frac{\sigma_\epsilon}{\phi_\pi - 0.8}$ is identified.
- Why? Joint likelihood density more concentrated on areas with higher values of ϕ_π and σ_ϵ.

Motivation - Estimation



(a) Impulse responses using prior setup 1



(b) Impulse responses using prior setup 2

- 1-unit monetary policy disturbance shock on inflation.
- Impulse response with two different priors (that has the same distribution over $(\rho, \frac{\sigma_\epsilon}{\phi-\rho})$).

Motivation - Policy Analysis

Suppose a central bank using the following small-sized model

$$y_t = \mathbb{E}_t [y_{t+1}] - \frac{1}{\sigma} (i_t - \mathbb{E}_t [\pi_{t+1}]) + g_t - \mathbb{E}_t [g_{t+1}]$$

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa (y_t - g_t) + u_t$$

$$i_t = \rho_R i_{t-1} + (1 - \rho_R) \psi_\pi \pi_t + (1 - \rho_R) \psi_y (y_t - g_t) + \varepsilon_{R,t}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t}, \quad g_t = \rho_g g_{t-1} + \varepsilon_{g,t}.$$

is trying to use the estimated parameter (history) $(\sigma, \beta, \kappa, \psi_\pi, \psi_y, \rho_R, \rho_g, \rho_u, \sigma_R, \sigma_g, \sigma_u)$, to choose a policy rule

$$i_t^* = \psi_\pi \pi_t + \psi_y (y_t - g_t)$$

that minimize welfare loss in the form of $\pi_t^2 + \alpha_x y_t^2$.

Motivation - Policy Analysis

- Now consider two policies $(\psi_\pi, \psi_y) = (1.5, 0)$, and $(1.5, 0.125)$

Table: Policy Comparison under Different Distributions and Weights

	$\frac{1}{\alpha_x} = 1$		$\frac{1}{\alpha_x} = 3$		$\frac{1}{\alpha_x} = 10$	
(ψ_π, ψ_y)	post 1	post 2	post 1	post 2	post 1	post 2
(1.5, 0)			✓		✓	✓
(1.5, 0.125)	✓	✓		✓		

- Policy choices are sensitive to prior choices as well.

Research Question

- Set-identification for parameters of interest.
  - ∗ sensitivity analysis: What's the identified set of parameters? How much can the posterior mean change as I change the prior?
- Given, for example, a DSGE model and observed data,
  - ∗ policy implications: Is it always possible to support a policy rule robust of priors?

Literature and Contributions

- **Robust Bayesian analysis**: Berger et al. (1994), Berger, Insua, and Ruggeri (2000), Gustafson (2015), *Giacomini and Kitagawa (2021)*, Ke, Montiel Olea, and Nesbit (2022), Giacomini, Kitagawa, and Read (2022)
- **Identification in DSGE models**: Canova and Sala (2009), Iskrev (2010), Komunjer and Ng (2011), Qu and Tkachenko (2012), Qu and Tkachenko (2017), Kociecki and Kolasa (2018), *Kocięcki and Kolasa (2023)*

Literature and Contributions

- **Frequentist inference for set-identified models:** Horowitz and Manski (2000), Manski (2003), Imbens and Manski (2004), Chernozhukov, Hong, and Tamer (2007), Stoye (2009), Romano and Shaikh (2010), Kaido, Molinari, and Stoye (2019)
- **Bayesian inference for set-identified models:** Baumeister and Hamilton (2015), Kline and Tamer (2016), Chen, Christensen, and Tamer (2018)
- **My contribution**:
  - A new Bayesian algorithm that can be applied to general structural models for estimation and inference.
  - I work on "global" identification rather than identification at certain point (KK23).
  - Method applied to DSGE models, whereas GK21's method only applicable to SVAR.

Estimate a Linearized DSGE model

Standard precedure

S 1. Summarize a macro model with equilibrium conditions, measurement equations, etc.

S 2. Log-linearization the equations around steady state, represent the model by a *linear rational expectation model* (LRM) with deep parameters θ.

$$\Gamma_0(\theta)\begin{bmatrix} S_t \\ P_t \end{bmatrix} = \Gamma_1(\theta)\mathbb{E}_t\begin{bmatrix} S_{t+1} \\ P_{t+1} \end{bmatrix} + \Gamma_2(\theta)S_{t-1} + \Gamma_3(\theta)\varepsilon_t$$

S_t state variables, P_t policy variables.

Estimate a Linearized DSGE model

Standard precedure

S 3. Solve the LREM, combine with measurement equations and attain a *state-space representation*.

$$S_t = A(\theta)S_{t-1} + B(\theta)\varepsilon_t$$
$$Y_t = C(\theta)S_{t-1} + D(\theta)\varepsilon_t$$

S 4. Use a generic filter to compute the likelihood $p(y \mid \theta)$ through the state-space model.

S 5. Start from a prior distribtuion π_θ, update by MCMC methods using the likelihood and obtain the posterior distribution of θ, $\pi_{\theta|y}$.

Setup

Assumption (1)

Linearized DSGE model with Gaussian shocks.

- Linear State-space representation

Assumption (2)

Solution to the LREM is unique, i.e. no indeterminacy.

- Coefficient of SS uniquely determined by solution.

Assumption (3)

Deep parameters enter LREM in an algebraic expression way.

- e.g. NKPC in Gali (2015): $\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \tilde{y}_t$

Definitions

Definition (OE)

Parameter $\bar{\theta}$ is observationally equivalent to θ if they yield the same data generating process.

- A property independent of data

Definition (Identification)

θ is identified if it has no observationally equivalent parameters.

- Define the equivalence mapping $K : \Theta \to 2^{\Theta}$, that is, $p(y \mid \theta) = p(y \mid \bar{\theta})$ for all y, if and only if $K(\theta) = K(\bar{\theta})$.

Algorithm

S.1 Run standard Bayesian estimation, get posterior draws of θ from a given prior π_θ.

*S.2** Optimize over the observationally equivalent set of parameters of this draw, find the lower and upper bounds of parameters of interest.

- Finding the OE set of a given parameter involves solving a polynomial system.

S.3 Average the lower/upper bounds for means and quantiles.

OE characterization

Assumptions

Define $N = APC' + B\Sigma D'$, where $P = E(S_t S_t')$,

Assumption (Stability)

For every $\theta \in \Theta$ and for any $z \in \mathbb{C}$, $\det(zI_{n_S} - A) = 0$ implies $| z | < 1$.

Assumption (Stochastic Minimality)

For every $\theta \in \Theta$, matrices $\mathcal{O}$ have full column rank and $\mathcal{C}$ have full row rank, i.e. $\text{rank}(\mathcal{O}) = \text{rank}(\mathcal{C}) = n_S$. Where $\mathcal{O} \equiv (C' \quad A'C' \quad \cdots \quad A'^{n_S-1}C')$, $\mathcal{C} \equiv (N \quad AN \quad \cdots \quad A^{n_S-1}N)$.

OE characterization

Theorem (KK23)

Let stability and stochastic minimality assumptions hold. Then $\theta \sim \bar{\theta}$ if and only if

1) $\bar{A} = TAT^{-1}$,

2) $\bar{C} = CT^{-1}$,

3) $AQA' - Q = T^{-1}\bar{B}\bar{\Sigma}\bar{B}'T'^{-1} - B\Sigma B'$,

4) $CQC' = \bar{D}\bar{\Sigma}\bar{D}' - D\Sigma D'$,

5) $AQC' = T^{-1}\bar{B}\bar{\Sigma}\bar{D}' - B\Sigma D'$,

for some nonsingular $n_\varepsilon \times n_\varepsilon$ *matrix* T *and symmetric* $n_\varepsilon \times n_\varepsilon$ *matrix* Q*. In addition, if* $\theta \sim \bar{\theta}$ *then both* T *and* Q *are unique.*

OE characterization

Brief

- In order to use KK23, given a parameter $\bar{\theta}$, we need to link it to the solutions.

- Attain the solution, $S_t = \bar{A}(\theta)S_{t-1} + \bar{B}(\theta)\varepsilon_t$ and $P_t = \bar{F}(\theta)S_{t-1} + \bar{G}(\theta)\varepsilon_t$, plug in LRM, equate coefficients on both sides in terms of S_{t-1}, and ϵ_t.

$$\bar{\Gamma}_0^s\bar{A} + \bar{\Gamma}_0^p\bar{F} - \bar{\Gamma}_1^s(\bar{A})^2 - \bar{\Gamma}_1^p\bar{F}\bar{A} = \bar{\Gamma}_2$$

$$\bar{\Gamma}_1^s\bar{A}\bar{B} + \bar{\Gamma}_1^p\bar{F}\bar{B} - \bar{\Gamma}_0^s\bar{B} + \bar{\Gamma}_3 = \bar{\Gamma}_0^p\bar{G}$$

OE characterization

Brief

Therefore, we can solve for observationally equivalent $\bar{\theta}$ following the procedure

S.1 Given θ, solve for state-space coefficients.

S.2 Characterize $\bar{\theta}$ by KK23 and the previous equations, unknowns include (not limit to) $\bar{\theta}$, $\bar{B}$, $\bar{G}$.

S.3 Reduce a polynomial system to its reduced Grobner basis.

Gröbner Basis

A reduced Gröbner basis is a set of *multivariate polynomials* enjoying certain properties that allow simple algorithmic solutions. For example, the equations:

$$x^3 - 2xy, \quad x^2 - 2y^2 + x.$$

has a reduced Gröbner basis

$$x^2, \quad xy, \quad y^2 - \frac{x}{2}.$$

- Any zero of a Gröbner basis is also a zero of the original system.
- Reduced Gröbner bases are unique for any given set of polynomials and any monomial ordering.

Additional Assumptions

Assumption (Measurability)

The equivalence mapping K is Effros measurable, that is,

$K^{-}(F) \equiv \{\theta : K(\theta) \cap F \neq \emptyset\} \in \mathcal{A}$ *for each* $F \in \mathcal{F}$.

Assumption (Continuity)

(1) *K is a continuous correspondence at θ_0.*

(2) *Parameters of interest $\eta : \Theta \to \eta(\Theta)$ is continuous.*

Assumption (Regularity)

Let the prior of deep parameters θ, π_θ, be absolutely continuous with respect to a σ-finite measure on $(\Theta, \mathcal{A})$, and $\pi_\theta(\Theta) = 1$.

Multiple priors

- Define the class of all priors that the marginal distribution for K coincides with the given π_K, i.e.,

$$\Pi_\theta(\pi_K) = \{\pi_\theta : \pi_\theta\left(\{\theta : K(\theta) \in B\}\right) = \pi_K(B), \text{ for } B \in \mathcal{B}(\mathcal{F})\}$$

* The class of priors induce the same prior predictive distribution $p(y) = \int p(y \mid \theta) d\pi_\theta$.

* The class of priors have the same push-forward measure π_K.

Robust Distributions

Theorem (Posterior Mean)

For a given π_θ, let measurability and regularity assumptions hold, that is, given a prior π_θ absolutely continuous with respect to a σ-finite measure, we have a push-forward measure π_K of π_θ under K that is also absolutely continuous. Define

$$\overline{\eta}^*(\theta) = \sup_{\theta' \in K(\theta)} \eta(\theta'), \quad \underline{\eta}^*(\theta) = \inf_{\theta' \in K(\theta)} \eta(\theta')$$

Then, the set of posterior means is characterized by

$$\sup_{\pi_{\theta|Y} \in \Pi_{\theta|Y}} \mathbb{E}_{\theta|Y}[\eta(\theta)] = \mathbb{E}_{\theta|Y}[\overline{\eta}^*(\theta)], \quad \inf_{\pi_{\theta|Y} \in \Pi_{\theta|Y}} \mathbb{E}_{\theta|Y}[\eta(\theta)] = \mathbb{E}_{\theta|Y}\left[\underline{\eta}^*(\theta)\right]$$

where $\Pi_{\theta|Y}$ collects the posteriors of $\Pi_\theta(\pi_K)$ for a given π_K.

Robust Distributions

Definitions for the proof

In random set theory (Molchanov and Molinari (2018)),

Definition (Selection)

Let $X : \Phi \rightrightarrows \mathcal{H}$ be a closed random set defined on the probability space $(\Phi, \mathcal{B}, \pi_{\phi|Y})$, and $\xi(\phi) : \Phi \to \mathcal{H}$ be its measurable selection, i.e., $\xi(\phi) \in X(\phi), \pi_{\phi|Y}$-a.s. Let $S^1(X)$ be the class of integrable measurable selections, $S^1(X) = \{\xi : \xi(\phi) \in X(\phi), \pi_{\phi|Y}\text{-a.s., } E_{\phi|Y}(\|\xi\|) < \infty\}$.

Definition (Aumann expectation)

The Aumann expectation of X is defined as $E^A_{\phi|Y}(X) \equiv \{E_{\phi|Y}(\xi) : \xi \in S^1(X)\}$

Robust Distributions

Proof

- First show that $\sup_{\pi_{\theta|Y} \in \Pi_{\theta|Y}} \mathbb{E}_{\theta|Y}[\eta(\theta)] = \mathbb{E}_{K|Y}\left[\sup_{\{\theta \in \Theta : \theta \in K\}} \eta(\theta)\right]$
- Note that

$$\left\{\mathbb{E}_{\theta|K}[\eta(\theta)] : \pi_{\theta|K}(K(\theta)) = 1\right\} = co\left(\{\eta(\theta), \theta \in K\}\right)$$

- A selection $\pi_{\theta|K}$ can be viewed as a selection from $co\left(\{\eta(\theta), \theta \in K\}\right)$.
- The set $\left\{E_{\theta|Y}(\eta(\theta)) = E_{K|Y}\left[E_{\theta|K}(\eta(\theta))\right] : \pi_{\theta|K} \in \Pi_{\theta|K}\right\}$ agrees with $E^{A}_{K|Y}\left[co\left(\{\eta(\theta), \theta \in K\}\right)\right]$ by the definition of the Aumann integral.
- Let the selection be $\sup_{\{\theta \in \Theta : \theta \in K\}} \eta(\theta)$ then its done.

Robust Distributions

Proof

- Then, show that $\mathbb{E}_{K|Y}\left[\sup_{\{\theta\in\Theta:\theta\in K\}}\eta(\theta)\right] = \mathbb{E}_{\theta|Y}\left[\overline{\eta}^*(\theta)\right]$
- Write out the expectation to integration, we have LHS equals

$$\int_{2^\Theta} \sup_{\{\theta\in\Theta:\theta\in K\}} \eta(\theta)\mathrm{d}\pi_{K|Y} = \int_{\Theta} \sup_{\{\theta'\in\Theta:\theta'\in K(\theta)\}} \eta(\theta')\mathrm{d}\pi_{\theta|Y},$$

where the second equality follows from a change of variable.

- RHS is

$$\mathbb{E}_{\theta|Y}\left[\bar{\eta}^*(\theta)\right] = \int_{\Theta} \bar{\eta}^*(\theta)\mathrm{d}\pi_{\theta|Y},$$

which equals to LHS because by definition,

$$\overline{\eta}^*(\theta) = \sup_{\theta'\in K(\theta)} \eta(\theta')$$

Robust Distributions

Theorem (Consistency)

Let, in addition continuity assumption hold, assume further that induced prior π_K leads to a consistent posterior, and $\Theta \subset \mathbb{R}^p$, $H \subset \mathbb{R}^q$ for some $p, q < \infty$ are compact spaces. Then the Hausdorff distance between the set of posterior means and the convex hull of true identified set goes to zero almost surely as T increases, i.e.,

$$\lim_{T\to\infty} d_H\left(E_{\theta|Y^T}\left[\left(\underline{\eta}^*(\theta), \bar{\eta}^*(\theta)\right)\right], \left(\underline{\eta}^*(\theta_0), \bar{\eta}^*(\theta_0)\right)\right) \to 0, \quad p(Y^\infty \mid \theta_0)\text{-a.s.}$$

Inferenece

Assumption (Convergence)

(i) *Let $\hat{\theta}$ denote an element of the set of maximum likelihood estimators,*

$$\sqrt{T}\begin{pmatrix}\underline{\eta}^*(\theta)-\underline{\eta}^*(\hat{\theta})\\ \overline{\eta}^*(\theta)-\overline{\eta}^*(\hat{\theta})\end{pmatrix} \mid Y^T \quad \Rightarrow \quad \mathcal{N}(0,\Sigma),\ \text{as } T\to\infty, p\left(Y^\infty \mid \theta_0\right)\text{-a.s.} \tag{1}$$

$$\sqrt{T}\begin{pmatrix}\underline{\eta}^*(\hat{\theta})-\underline{\eta}^*(\theta_0)\\ \overline{\eta}^*(\hat{\theta})-\overline{\eta}^*(\theta_0)\end{pmatrix} \mid \theta_0 \quad \Rightarrow \quad \mathcal{N}(0,\Sigma),\ \text{as } T\to\infty. \tag{2}$$

(ii) *For the robust credible region* $\left[\underline{q}^*_{\alpha/2}, \overline{q}^*_{1-\alpha/2}\right]$,

$$\hat{c}_T \equiv \sqrt{T}\begin{pmatrix}\underline{q}^*_{\alpha/2}-\underline{\eta}^*(\hat{\theta})\\ \overline{q}^*_{1-\alpha/2}-\overline{\eta}^*(\hat{\theta})\end{pmatrix} \xrightarrow{p} c \tag{3}$$

for some constant c as $T\to\infty$.

Inference

Theorem (Coverage)

Under some regularity assumptions and Assumption Convergence,

$$\liminf_{T\to\infty} P_{Y^T|\theta_0}\left(\eta(K(\theta_0)) \subset \left[\underline{q}^*_{\alpha/2}, \overline{q}^*_{1-\alpha/2}\right]\right) \geq 1-\alpha. \quad (4)$$

Example 1: Cochrane Model

Consider the full model

$$x_t = \rho x_{t-1} + \epsilon_t, \quad |\rho| < 1, \epsilon_t \sim N(0, \sigma_\epsilon)$$

$$i_t = r + E_t \pi_{t+1}$$

$$i_t = r + \psi \pi_t + x_t, \quad \psi > 1$$

Structural parameters are $\theta = (\rho, \psi, \sigma_\epsilon)$. The solution is equivalent to a AR(1) setting

$$\pi_t = \rho \pi_{t-1} + \frac{1}{\psi - \rho} \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

with $\phi = (\rho, \frac{\sigma_\epsilon}{\psi - \rho})$

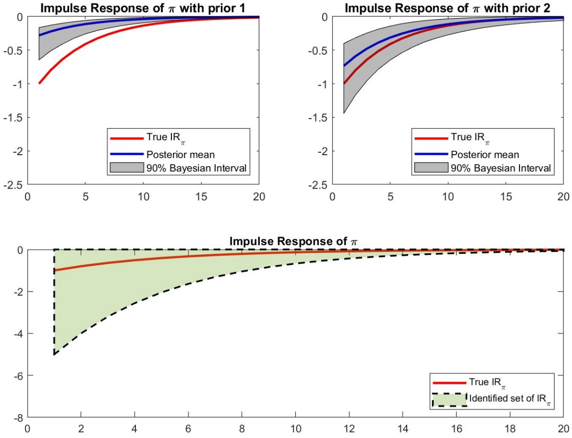
Example 1: Inference

Table: Estimated Identified Set

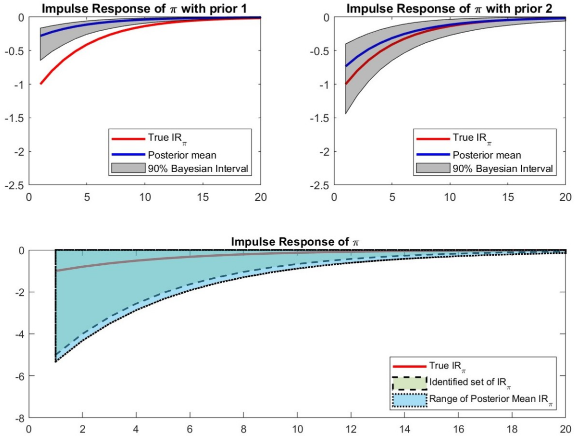
	True value	Identified set	Range of posterior mean
σ_e	1	$(0.2, \infty)$	$(0.21, \infty)$
ϕ_π	1.8	$(1, \infty)$	$(1.00, \infty)$
ρ	0.8	0.8	0.80

- Estimation of range of posterior means approximates the identified set well.

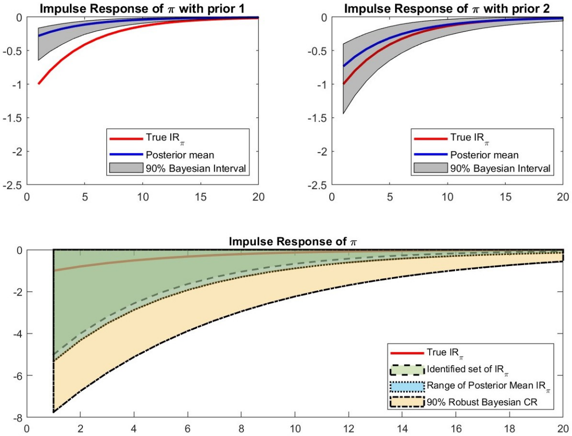
Example 1: Inference



Example 1: Inference



Example 1: Inference



Example 2: Three-equation New Keynesian

Consider the following model,

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) + \varepsilon_{yt}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + \varepsilon_{\pi t}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t\right) + \varepsilon_{Rt}$$

$$\varepsilon_{jt} \sim N(0,1); \quad j = y, \pi, R$$

where π_t inflation, y_t the output gap, i_t the nominal interest rate. $\kappa = \frac{(1-\tau)(1-\beta\tau)}{\tau}(\sigma + \psi)$ is the slope of the Philips curve. Since (ψ, τ) only enter the equation system via κ, ψ can be jointly unidentified with τ. The identification problem here is purely mathematical and is almost trivial.

Example 2: An and Shorfheide (2007)

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] + \mathbb{E}_t[z_{t+1}]) + g_t - \mathbb{E}_t[g_{t+1}]$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa(y_t - g_t)$$

$$i_t = \rho_R i_{t-1} + (1-\rho_R)\psi_\pi \pi_t + (1-\rho_R)\psi_y(y_t - g_t) + \varepsilon_{R,t}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \quad g_t = \rho_g g_{t-1} + \varepsilon_{g,t}.$$

- $(\psi_\pi, \psi_y, \rho_R, \sigma_R)$ are not identified.
- All the shocks, either has no effect on π_t or y_t, or affect π_t and y_t in the same direction.

Example 2: A Cost-push Shock Model

To generate meaningful trade-off between output gap and inflation,

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}]) + g_t - \mathbb{E}_t[g_{t+1}]$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa(y_t - g_t) + u_t$$

$$i_t = \rho_R i_{t-1} + (1-\rho_R)\psi_\pi \pi_t + (1-\rho_R)\psi_y(y_t - g_t) + \varepsilon_{R,t}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t}, \quad g_t = \rho_g g_{t-1} + \varepsilon_{g,t}.$$

- Positive cost-push shock $u \longrightarrow y \downarrow, \pi \uparrow$
- Positive monetary policy shock $\epsilon_R \longrightarrow y \downarrow, \pi \downarrow$

Example 2: Policy

Table: Policy Comparison under Different Distributions and Weights

	$\frac{1}{\nu\kappa} = 1$		$\frac{1}{\nu\kappa} = 3$		$\frac{1}{\nu\kappa} = 10$	
(ψ_π, ψ_y)	post 1	post 2	post 1	post 2	post 1	post 2
(1.5, 0)			✓		✓	✓
(1.5, 0.125)	✓	✓		✓		
(1.5, 1)	✓	✓				
(5, 0)			✓	✓	✓	✓

Smets and Wouters 2007

	True value	Posterior mean	(Robust) Bayesian CR
$\alpha_1 = \frac{(\gamma-1+\delta)\alpha}{\beta^{-1}\gamma^{\sigma_c}-1+\delta}$	0.17	0.17	[0.16,0.18]
$\alpha_2 = \frac{\lambda\gamma^{-1}}{1+\lambda\gamma^{-1}}$	0.41	0.41	[0.41,0.42]
$\alpha_3 = \frac{(1-\alpha)(\sigma_c-1)}{\phi_w\sigma_c(1+\lambda\gamma^{-1})(1-\alpha_1-g_y)}$	0.13	0.13	[0.12,0.14]
$\alpha_4 = \frac{1-\lambda\gamma^{-1}}{(1+\lambda\gamma^{-1})\sigma_c}$	0.12	0.13	[0.13,0.13]
$\alpha_5 = \frac{1}{1+\beta\gamma^{1-\sigma_c}}$	0.50	0.50	[0.50,0.50]
$\alpha_6 = \frac{1}{(1+\beta\gamma^{1-\sigma_c})\varphi\gamma^2}$	0.09	0.09	[0.08,0.10]
$\alpha_7 = \beta\gamma^{-\sigma_c}(1-\delta)$	0.97	0.97	[0.97,0.97]
$\alpha_8 = (1-\delta)\gamma^{-1}$	0.97	0.97	[0.97,0.97]
$\alpha_9 = (1-\alpha_8)(1+\beta\gamma^{1-\sigma_c})\varphi\gamma^2$	0.29	0.31	[0.28,0.34]
$\alpha_{10} = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}$	0.19	0.16	[0.14,0.19]
$\alpha_{11} = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}$	0.80	0.83	[0.81,0.86]
$\alpha_{12} = \frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{(1+\beta\gamma^{1-\sigma_c}\iota_p)\xi_p[(\phi_p-1)\varepsilon_p+1]}$	0.02	0.02	[0.02,0.02]
$\alpha_{13} = \frac{1}{1-\lambda\gamma^{-1}}$	3.41	3.37	[3.29,3.44]
$\alpha_{14} = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}$	0.29	0.29	[0.27,0.31]
$\alpha_{15} = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\delta_c}}$	0.79	0.79	[0.77,0.81]
$\alpha_{16} = \frac{(1-\beta\gamma^{1-\sigma_c}\xi_w)(1-\xi_w)}{(1+\beta\gamma^{1-\sigma_c})\xi_w[(\phi_w-1)\varepsilon_w+1]}$	0.00	0.01	[0.01,0.01]

- Same setup as kk23. Same identification results, but "globally".

Conclusion

In this paper, I address the following issues:

- Estimation of highly structural set-identified models can be challenging. medskip
  - $*$ An Bayesian algorithm that is generally applicable is proposed.
- Estimation results of set-identified DSGE models are sensitive to choice of priors (Identification)
  - $*$ Use a robust Bayesian algorithm, I can pick any 'reasonable' prior and obtain robust results.
  - $*$ I also prove it asymptotically finds the frequentist identified set.
- Researchers are silent about non-identified DSGE models (Policy-implication)
  - $*$ The collection of posterior means of parameters of interest is given.
  - $*$ One may still have nontrivial conclusions even when the model suffers identification problems.