Individual and Common Information: Model-free Evidence from Probability Forecasts

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Individual and common information acquisition

Information can improve decisions taken under uncertainty

From the theoretical literature we know that:

- The marginal value of information is state-dependent
- Common information is more likely to affect aggregate outcomes
- Private vs public information dichotomy important in strategic settings

Little empirical work studying relative importance of individual vs common information outside highly structural models

This paper

What we do:

1. Propose a method to extract individual and common signals from repeated cross-section of probability forecasts under weak assumptions
2. Ask and answer new questions about the empirical properties of individual and common information

Key assumption: Forecasters use Bayes' rule to update their beliefs

The plan

1. The Survey of Professional Forecasters (SPF) probability forecasts
2. Extracting common and individual signals from a cross-section of belief revisions
3. Empirical evidence on the informativeness of individual and common signals
4. Characterize the estimated signals under alternative information structures

Related literature

Empirical papers using SPF survey data

- **Accuracy of SPF:** Zarnowitz (1979), Zarnowitz and Braun (1993), Diebold, Tay, and Wallis (1997), Clements (2006, 2018), Engelberg, Manski and Williams (2009) and Kenny, Kostka and Masera (2014).
- **Forecast combination:** Bonham and Cohen (2001) and Genre, Kenny, Meyler and Timmermann (2013).
- **Testing theories of expectations formation:** Zarnowitz (1985), Keane and Runkle (1990), Bonham and Dacy (1991), Laster, Bennett and Geoum (1999) and Coibion and Gorodnichenko (2012,2015).

Micro vs macro news

- Born, Enders, Menkhof, Mueller and Niemann (2022).

Structural macro models with public and private signals

- Nimark (2008), Lorenzoni (2009,2010), Melosi (2014), Nimark (2014), Chahrour, Nimark and Pitschner (2021).

Endogenous information acquisition

- Sims (1998, 2003), Mackowiack and Wiederholt (2009, 2015), Woodford (2009), Chiang (2022), Flynn and Sastry (2022)

The SPF data

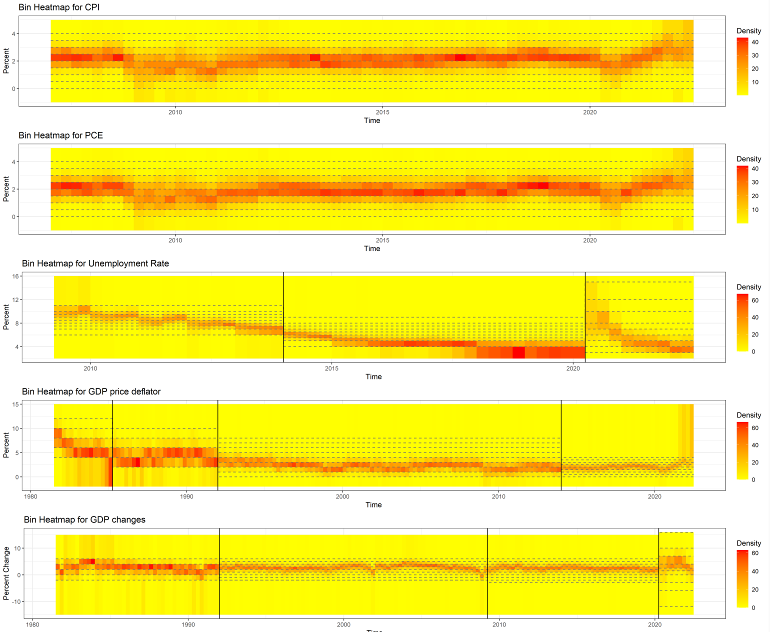
The Survey of Professional Forecasters

Quarterly survey of practitioners about macroeconomic variables

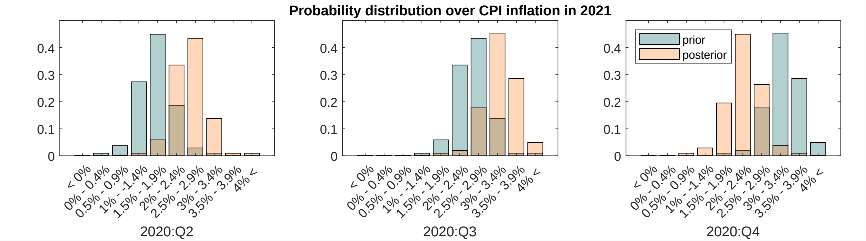
- Participants are from industry, Wall Street, commercial banks and academic research centers
- Survey elicits both point and probability forecasts
- Probability forecasts
    - GDP growth (1968:Q4 $\rightarrow$), GDP deflator (1968:Q4 $\rightarrow$), PCE (2007:Q1 $\rightarrow$), CPI (2007:Q1 $\rightarrow$) and unemployment (2009:Q2 $\rightarrow$)
    - Fixed-event forecasts about calendar year outcomes
    - Outcome bins pre-specified by administrators of survey
- Forecasters are anonymous to users of the survey but trackable through id numbers

Fixed-event forecasts allow us to observe how cross-section of beliefs about a given calendar year is revised over time

Heat map for average density forecasts



Example: Observed belief revisions of forecaster #570



Decomposing a cross-section of belief revisions

Decomposing a cross-section of belief revisions

Common signal

- *What is the single signal that, if observed by all forecasters, can explain the most of the belief revisions of all the forecasters?*

Individual signal

- *What is the signal that is necessary to explain a forecaster's residual belief revision not accounted for by the common signal?*

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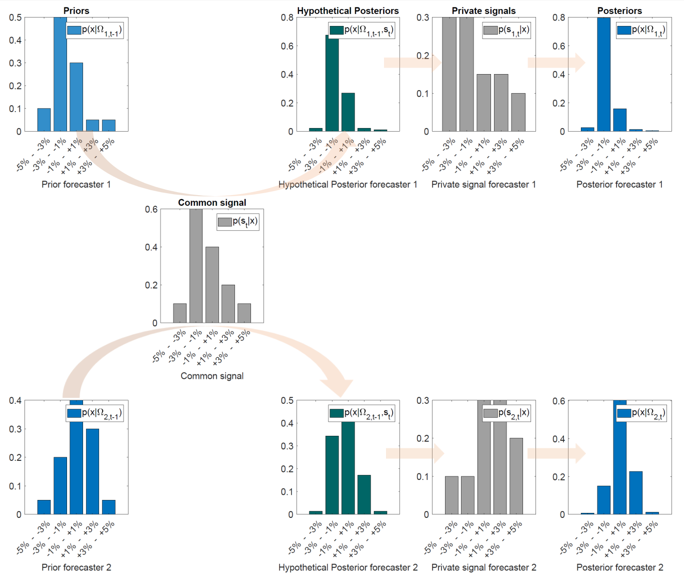
Signals and the cross-section of belief revisions



Notation

- Generic macroeconomic outcome $x_n \in X : n = 1, 2, ..., N$
- Forecasters indexed by $j = 1, 2, ..., J$
- Signals $s \in S$
- Prior beliefs of forecaster j is $p(x \mid \Omega^j_{t-1})$
- Posterior beliefs of forecaster j is $p(x \mid \Omega^j_t) = p(x \mid \Omega^j_{t-1}, s_t, s^j_t)$

Bayes rule, belief updates and realized signals

Bayes' rule give the posterior probability of x_n as

$$p(x_n \mid \Omega^j_{t-1}, s_t) = \frac{p(s_t \mid x_n)p(x_n \mid \Omega^j_{t-1})}{p(s_t \mid \Omega^j_{t-1})}.$$

Since $p(s_t)$ is a normalizing constant independent of x we get

$$p(s_t \mid x_n) \propto \frac{p(x_n \mid \Omega^j_{t-1}, s_t)}{p(x_n \mid \Omega^j_{t-1})}.$$

Note:

- From now on, a **signal** means $p(s \mid x) \in [0,1]^N$
- Signal labels do not matter for how agents update their beliefs
- An observed belief revision is informative about the properties of the realized signal, not the complete signal structure $p(S \mid X)$

Defining the common signal

The estimated **common signal** $\widehat{s}_t$ about the event x is defined as

$$\widehat{s}_t = \arg\min_{s \in [0,1]^N} \sum_{j=1}^{J} KL(\Omega_t, \Omega_{t-1}, s_t)$$

where $KL(\Omega_t, \Omega_{t-1}, s_t)$ is the Kullback-Leibler divergence

$$KL(\Omega_t^j, \Omega_{t-1}^j, s_t) = \sum_{n=1}^{N} p(x_n \mid \Omega_t^j) \log\left(\frac{p(x_n \mid \Omega_t^j)}{p(x_n \mid \Omega_{t-1}^j, s_t)}\right).$$

- $p(x \mid \Omega_t^j)$ = observed posterior
- $p(x \mid \Omega_{t-1}^j, s_t)$ = beliefs induced by s_t

Inverting Bayes Rule to extract individual signals

Define the **individual signal** s_t^j as the signal that when combined with the common signal and the observed prior result in the observed posterior.

From Bayes' rule

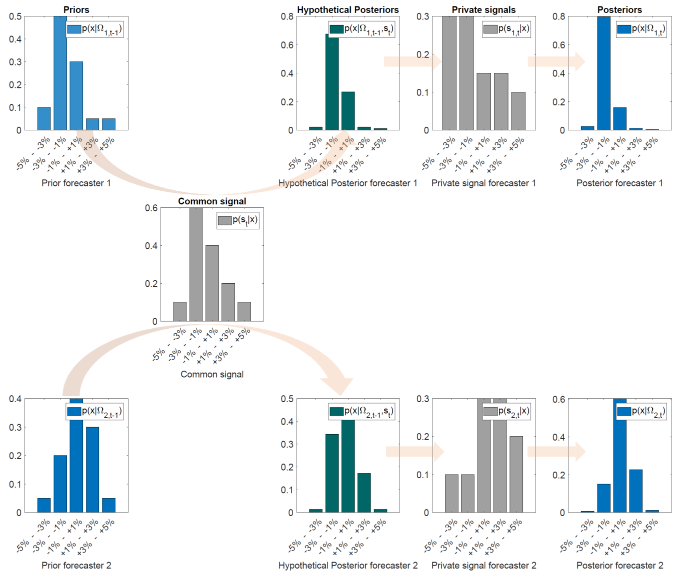
$$p(x_n \mid \Omega_{t-1}^j, s_t, s_t^j) = \frac{p(s_t^j \mid x_n) p(x_n \mid \Omega_{t-1}^j, s_t)}{p(s_t^j \mid \Omega_{t-1}^j, s_t)}.$$

so that

$$p(s_t^j \mid x_n) \propto \frac{p(x_n \mid \Omega_{t-1}^j, s_t, s_t^j)}{p(x_n \mid \Omega_{t-1}^j, s_t)}.$$

where $p(x \mid \Omega_t^j) \equiv p(x_n \mid \Omega_{t-1}^j, s_t, s_t^j)$ is the period t posterior.

Signals and the cross-section of belief revisions



3 measures of signal informativeness

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1. The **update measure** captures magnitude of belief revision

$$KL(s, \Omega^j) = \sum_{n=1}^{N} p(x_n \mid \Omega^j) \log \left(\frac{p(x_n \mid \Omega^j)}{p(x_n \mid \Omega^j, s)} \right)$$

2. The **negative entropy measure** captures magnitude of belief revision from a maximum entropy prior

$$H(s) = \sum_{n=1}^{N} p\left(x_n \mid \Omega^u, s\right) \log p\left(x_n \mid \Omega^u, s\right)$$

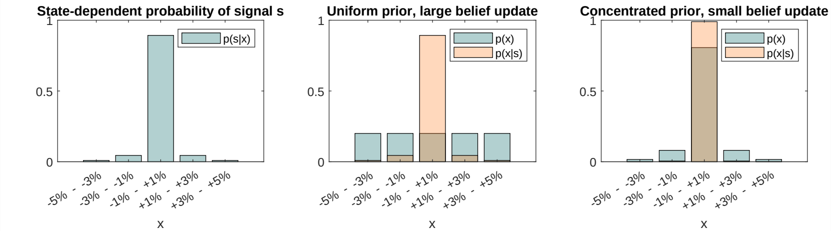
   where Ω^u is the uniform prior.

3. The **precision measure** captures precision of signal

$$P(s) = var\left(x_n \mid \Omega^u, s\right)^{-1}$$

All measures are defined so that a higher value implies a more informative signal

The update measure and the prior



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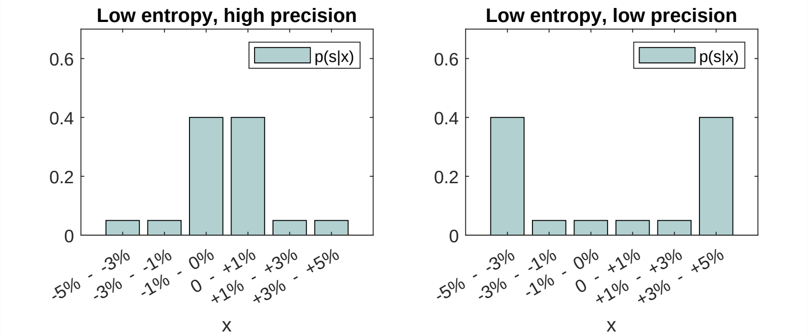
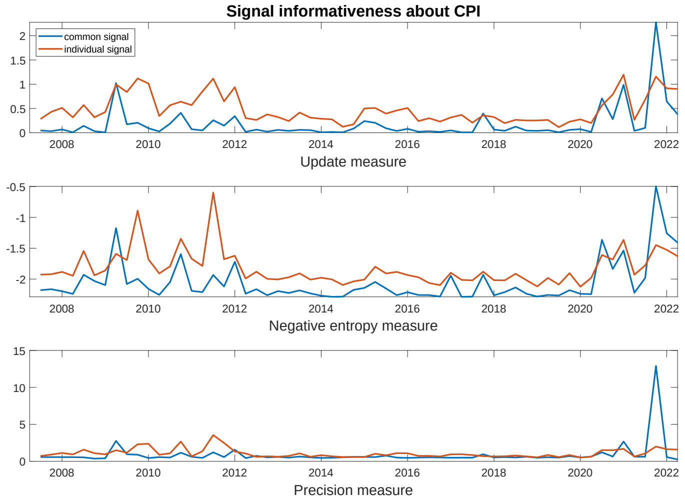
Entropy vs variance measures

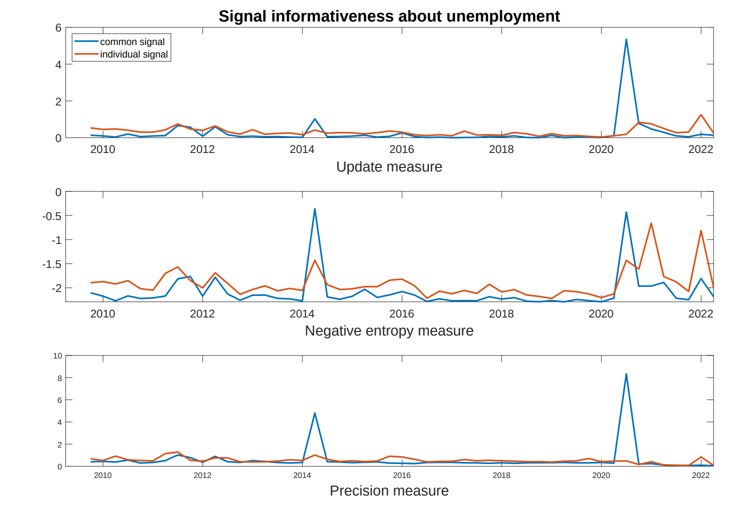


Empirical properties of individual and common signals

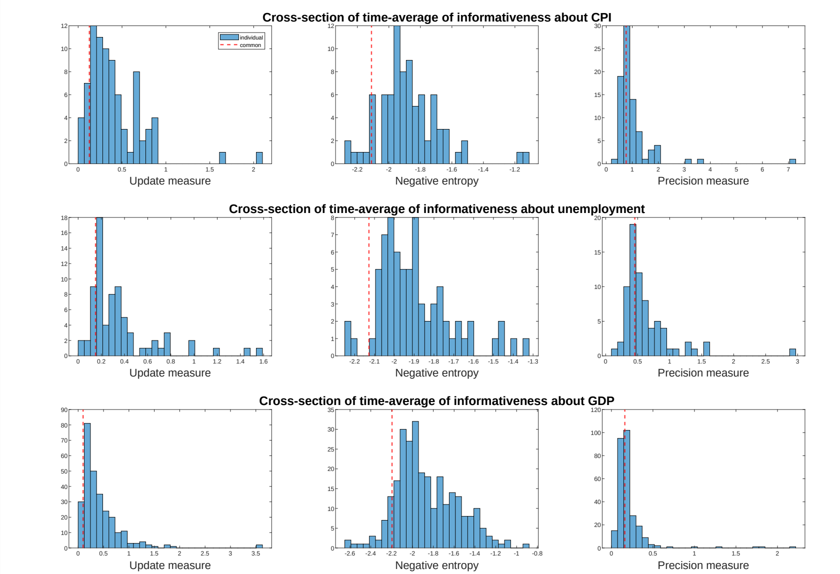
Time varying informativeness of signals about CPI inflation



Time varying informativeness of signals about unemployment



Cross-section of informativeness of signals



Informativeness and macro outcomes: CPI inflation

CPI inflation

	π_t^{cpi}	π_{t-1}^{cpi}	$\Delta\pi_t^{cpi}$	$\lvert\Delta\pi_t^{cpi}\rvert$	$\lvert\Delta\pi_{t-1}^{cpi}\rvert$
Individual signals					
KL	-0.08	-0.13	0.08	0.48	0.45
H	-0.20	-0.22	-0.03	0.36	0.35
P	-0.17	-0.22	0.05	0.36	0.35
Common signals					
KL	0.12	0.15	-0.03	0.23	0.44
H	0.25	0.21	0.14	0.45	0.53
P	0.02	0.04	-0.12	-0.06	0.29

Table 1: Correlation of information measures and CPI inflation outcomes.

Informativeness and macro outcomes: CPI inflation

CPI inflation

	π_t^{cpi}	π_{t-1}^{cpi}	$\Delta\pi_t^{cpi}$	$\lvert\Delta\pi_t^{cpi}\rvert$	$\lvert\Delta\pi_{t-1}^{cpi}\rvert$
Individual signals					
KL	-0.08	-0.13	0.08	0.48	0.45
H	-0.20	-0.22	-0.03	0.36	0.35
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Common signals					
KL	0.12	0.15	-0.03	0.23	0.44
H	0.25	0.21	0.14	0.45	0.53
P	0.02	0.04	-0.12	-0.06	0.29

Table 2: Correlation of information measures and CPI inflation outcomes.

Informativeness and macro outcomes: Unemployment

	Unemployment				
	u_t	u_{t-1}	Δu_t	$\|\Delta u_t\|$	$\|\Delta u_{t-1}\|$
			Individual signals		
KL	0.27	0.38	-0.18	-0.06	-0.19
H	0.16	0.31	-0.24	0.07	-0.10
P	0.32	0.28	0.06	-0.11	-0.11
			Common signals		
KL	0.22	0.48	-0.41	0.38	0.14
H	0.20	0.40	-0.31	0.24	0.04
P	0.21	0.43	-0.35	0.31	0.12

Table 3: Correlation of information measures and unemployment outcomes.

Informativeness and macro outcomes: Unemployment

	Unemployment				
	u_t	u_{t-1}	Δu_t	$\lvert\Delta u_t\rvert$	$\lvert\Delta u_{t-1}\rvert$
Individual signals					
KL	0.27	0.38	-0.18	-0.06	-0.19
H	0.16	0.31	-0.24	0.07	-0.10
P	0.32	0.28	0.06	-0.11	-0.11
Common signals					
KL	0.22	0.48	-0.41	0.38	0.14
H	0.20	0.40	-0.31	0.24	0.04
P	0.21	0.43	-0.35	0.31	0.12

Table 4: Correlation of information measures and unemployment outcomes.

Some implications for theoretical models

Informativeness and the business cycle: Theory

Information counter-cyclical: Incentives to acquire information strongest during downturns

- Chiang (WP 2022), Song and Stern (2022) and Flynn and Sastry (WP 2022)

or

Information pro-cyclical: Economic activity generates information

- Chalkley and Lee (RED 1998), Veldkamp (JET 2005), Van Nieuwerburgh and Veldkamp (JEEA 2006), Ordoñez (JPE 2013), Fajgelbaum, Shaal and Taschereau-Dumouchel (QJE 2017)

The Anxious Index:
Informativeness and probability of a recession

	CPI inflation	*unemployment*	*GDP growth*	*GDP deflator*	*PCE inflation*
			Individual signals		
KL	0.20	0.06	0.27	0.23	0.24
H	0.15	0.24	0.27	0.17	0.24
P	0.13	-0.20	-0.02	-0.06	0.23
			Common signals		
KL	0.16	0.72	0.18	0.08	0.19
H	0.26	0.45	0.24	0.14	0.17
P	0.03	0.58	0.04	-0.10	0.04

Table 5: Correlation between the Philadelphia Fed's *Anxious Index* and the measures of informativeness.

But: Informativeness of signals only weakly correlated with NBER recessions and with mixed signs.

The VIX Index: Informativeness and financial volatility

	CPI inflation	*unemployment*	*GDP growth*	*GDP deflator*	*PCE inflation*
			Individual signals		
KL	0.29	0.36	0.25	0.12	0.22
H	0.29	0.30	0.20	0.10	0.23
P	0.32	0.03	0.17	-0.02	0.19
			Common signals		
KL	0.12	0.26	0.22	0.15	0.17
H	0.25	0.16	0.22	0.12	0.22
P	0.02	0.10	0.17	-0.07	0.05

Table 6: Correlation between VIX and measures of informativeness.

Characterizing the extracted signals

A sufficient FOC for the common signal

Proposition. The estimated common signal $\widehat{s}_t$ induces average beliefs equal to the average observed posterior distribution

$$\frac{1}{J}\sum_{j=1}^{J} p\left(x_n \mid \Omega_{t-1}, \widehat{s}_t\right) = \frac{1}{J}\sum_{j=1}^{J} p\left(x_n \mid \Omega_t\right) : n = 1, 2, ..., N.$$

Corollary. The estimated individual signals induces belief updates that average to zero across agents

$$\frac{1}{J}\sum_{j=1}^{J}\left[p\left(x_n \mid \widehat{s}_t^j, \widehat{s}_t, \Omega_{t-1}^j\right) - p\left(x_n \mid \widehat{s}_t, \Omega_{t-1}^j\right)\right] = 0 : n = 1, 2, ..., N.$$

The mean-posterior-over-mean-prior-odds ratio and the common signal

The mean-posterior-over-mean-prior odds ratio R_m^n is defined as

$$R_m^n = \left(\frac{\frac{1}{J}\sum_{j=1}^{J} p\left(x_n \mid \Omega_t^j\right)}{\frac{1}{J}\sum_{j=1}^{J} p\left(x_m \mid \Omega_t^j\right)} \right) / \left(\frac{\frac{1}{J}\sum_{j=1}^{J} p\left(x_n \mid \Omega_{t-1}^j\right)}{\frac{1}{J}\sum_{j=1}^{J} p\left(x_m \mid \Omega_{t-1}^j\right)} \right)$$

The ratio R_m^n captures how much period t information shifts average beliefs in favor of state n relative to state m.

Proposition. If the prior beliefs of all forecasters coincide, the relative probability of observing $\widehat{s}_t$ in states n and m is given by

$$\frac{p\left(\widehat{s}_t \mid x_n\right)}{p\left(\widehat{s}_t \mid x_m\right)} = R_m^n.$$

Conditions for convergence to true signal in limit with large number of agents

Proposition. Let $p\left(s^j \mid x_n\right)$ be a random variable with support $(0,1)$ and mean μ_n^j. The estimated signal converges in probability to the true common signal, i.e. $\hat{s} \rightarrow s$ as $J \rightarrow \infty$, if $\mu_m^j = \mu_n^k$ for each $m, n \in \{1, 2, ..., N\}$ and $j, k \in \{1, 2, ..., J\}$ and if $p\left(s^j \mid x_n\right)$ is independent of $p\left(x_m \mid s_t, \Omega_{t-1}^j\right)$ for each $m, n \in \{1, 2, ..., N\}$.

These conditions are **very stringent**: Rules out that individual signals are on average informative

A popular linear-Gaussian information structures

Priors $x \mid \Omega^j_{t-1} \sim N\left(\underline{\mu}^j, \underline{\sigma}^2\right)$ where $\underline{\mu}^j \sim N\left(\underline{\mu}, \sigma^2_\mu\right)$.

Common signal $s_t = x + \eta : \eta \sim N\left(0, \sigma^2_\eta\right)$

Individual signal $s^j_t = x + \varepsilon^j : \varepsilon^j \sim N\left(0, \sigma^2_\varepsilon\right)$

Posterior of agent j

$$E\left(x \mid \Omega^j_{t-1}, s_t, s^j_t\right) = g_\mu \underline{\mu}^j + g_s s_t + g_j s^j_t$$

$$var\left(x \mid \Omega^j_{t-1}, s_t, s^j_t\right) = \left(\underline{\sigma}_j^{-2} + \sigma_\eta^{-2} + \sigma_\varepsilon^{-2}\right)^{-1}$$

where

$$g_\mu = \frac{\underline{\sigma}^{-2}}{\underline{\sigma}^{-2} + \sigma_\eta^{-2} + \sigma_\varepsilon^{-2}}, g_s = \frac{\sigma_\eta^{-2}}{\underline{\sigma}^{-2} + \sigma_\eta^{-2} + \sigma_\varepsilon^{-2}}, g_j = \frac{\sigma_\varepsilon^{-2}}{\underline{\sigma}^{-2} + \sigma_\eta^{-2} + \sigma_\varepsilon^{-2}}.$$

Linear-Gaussian information structures

Proposition. Up to the discrete approximation, the estimated common signal $\hat{s}$ has conditional distribution

$$p\left(\hat{s} \mid x\right) \sim N\left(\widehat{x}, \hat{\sigma}_{\eta}^{-2}\right)$$

where

$$\widehat{x} = (1 - \widehat{g})^{-1}\left[\left(g_{\mu} - \widehat{g}\right)\underline{\mu} + g_s s + g_j x\right]$$

for $\widehat{g} = \frac{\underline{\sigma}^{-2}}{\hat{\sigma}_{\eta}^{-2} + \underline{\sigma}^{2}}$ and where $\sigma_{\hat{\eta}}^{-2}$ solves the equation

$$g_{\mu}^2 \sigma_{\underline{\varepsilon}}^2 + g_j^2 \sigma_{\varepsilon}^2 + \left(\underline{\sigma}^{-2} + \sigma_{\eta}^{-2} + \sigma_{\varepsilon}^{-2}\right)^{-1} = \hat{g}^2 \sigma_{\underline{\varepsilon}}^2 + \left(\underline{\sigma}^{-2} + \hat{\sigma}_{\eta}^{-2}\right)^{-1}.$$

Linear-Gaussian information structures

Corollary. The estimated common signal $\hat{s}_t$ coincides with s for all realizations if and only if $\sigma_\varepsilon^2 \to \infty$.

Corollary. If the true common signal is uninformative $\left(\sigma_\eta^2 \to \infty\right)$, then the estimated common signal is of the form $\widehat{s} = \alpha\left(x - \mu\right)$ with $\alpha \geq 1$.

Corollary. The estimated common signal precision $\hat{\sigma}_\eta^{-2}$ is increasing in both σ_ε^{-2} and σ_η^{-2} .

Corollary. The estimated private signals $\hat{s}^j \sim N\left(g\underline{\mu}^j + g_s s + g_j x, \hat{\sigma}_\varepsilon^2\right)$ where

$$\hat{\sigma}_\varepsilon^{-2} = \sigma_\varepsilon^{-2} - \left(\hat{\sigma}_\eta^{-2} - \sigma_\eta^{-2}\right)$$

Summing up

Decompose cross-section of belief revisions into common and idiosyncratic sources

- Method imposes only relatively weak assumptions
- Individual signals on average more informative than common signals
    - Large heterogeneity across forecasters
- Informativeness of both individual and common signals about macro outcomes increase when recession probability is high
    - Information acquisition appears to be counter-cyclical
- Characterized properties of extracted signals in alternative settings
    - Allows for model dependent interpretations
    - Method provides upper bound for importance of common signal

Common signal, different models

In a rational expectations model, all agents have model consistent expectations and hence share the same model.

If different agents use different models, agent j's posterior is given by

$$p(x \mid \Omega^j_{t-1}, s_t) = \frac{p_j(s_t \mid x) p(x \mid \Omega^j_{t-1})}{p(s_t \mid \Omega^j_{t-1})}$$

Proposition. With agent specific likelihood functions but a common prior, the estimated common signal satisfies

$$\frac{p\left(\widehat{s}_t \mid x_n\right)}{p\left(\widehat{s}_t \mid x_m\right)} = \frac{\frac{1}{J}\sum_{j=1}^{J} p_j(s_t \mid x_n)}{\frac{1}{J}\sum_{j=1}^{J} p_j(s_t \mid x_m)}$$

for each pair $n, m \in 1, 2, ..., N$.

Time varying informativeness of signals about GDP growth