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Robust Bayesian Estimation and Inference for Dynamic Stochastic General Equilibrium Models

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Motivation

- **DSGE models are widely used:**
  - U.S. Fed, Bank of Canada, Sveriges Riksbank etc.
- **Analysis of the models is challenging because of 'identification':**
  - DSGE models are micro-founded, rich with parameters.
  - Multiple parameter combinations may yield same data generating process.
  - Standard Bayesian methods can be sensitive to prior choices.

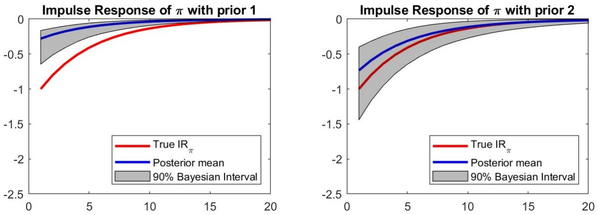
Motivation

A monetary policy model (Cochrane 2011, JPE). In its AR(1) form

$$\pi_t = \rho \pi_{t-1} + \frac{1}{\psi - \rho}\epsilon_t, \quad \psi > 1, |\rho| < 1, \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

parameter vector $(\psi, \sigma_\epsilon, \rho)$, Taylor rule parameter ψ, monetary policy disturbance coefficient ρ, its standard error σ_ϵ. Inflation rate π_t is observed.

here



Research Question

- Given a DSGE model and observed data.
    - ∗ Sensitivity analysis: How much can the results change as I change the prior?
    - ∗ Policy implications: Is there a way to provide the entire set of parameters of interest, robust of priors?

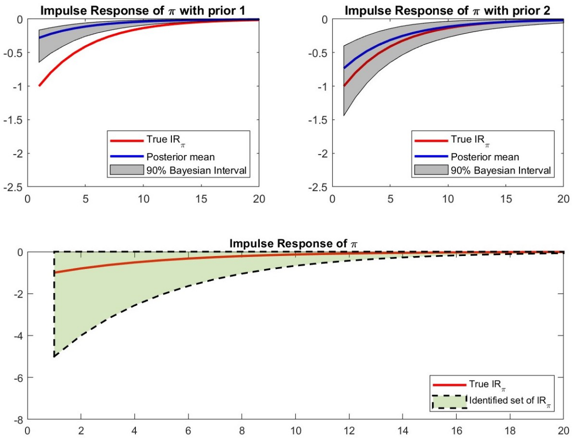
- Overview of the algorithm:

S.1 Run standard Bayesian estimation, get posterior draws of θ from a given prior $p(\theta)$.

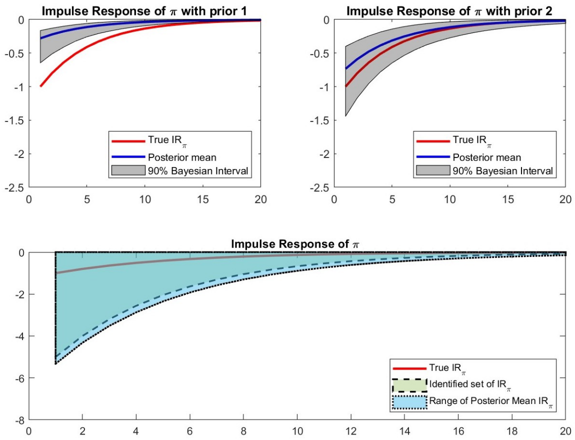
S.2* Optimize over the observationally equivalent set of parameters of this draw, find the lower and upper bounds of parameters of interest.

S.3 Average the lower/upper bounds for means and quantiles.

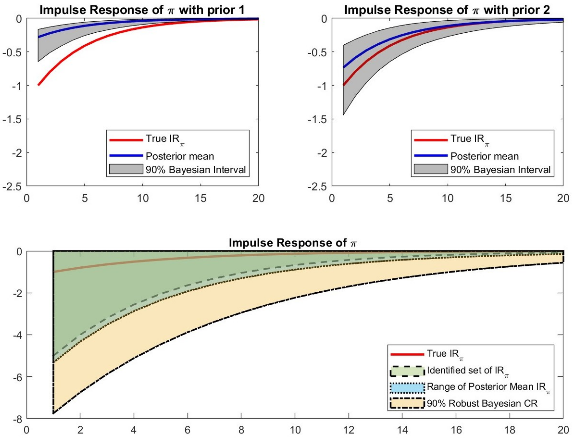
Preview of Results - Identification



Preview of Results - Inference



Preview of Results - Inference



Literature and Contributions

- **Identification in DSGE models**: Canova and Sala (2009), Iskrev (2010), Komunjer and Ng (2011),Qu and Tkachenko (2012), Qu and Tkachenko (2017), Kociecki and Kolasa (2018), Kociecki and Kolasa (2021)
- **Robust Bayesian analysis**: Berger et al. (1994), Berger, Insua, and Ruggeri (2000),Gustafson (2009), Giacomini and Kitagawa (2021), Ke, Montiel Olea, and Nesbit (2022), Giacomini, Kitagawa, and Read (2022)
- **My contribution**:
  - A robust Bayesian algorithm for DSGE models that is easy to implement and has a strong theoretical foundation.
  - I work on "global" identification rather than identification at certain point (KK21).
  - I study DSGE model, which has further complications (different with GK21).

Model Assumptions

Assumption (1)

Linearized DSGE model with Gaussian shocks.

Assumption (2)

Solution to the LREM is unique, i.e. no indeterminacy.

Assumption (3)

Deep parameters enter LREM in an algebraic expression way.

- e.g. NKPC in Gali (2015): $\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \tilde{y}_t$

Definitions

Definition (OE)

Parameter $\bar{\theta}$ is observationally equivalent to θ if they have the same likelihood $p(y \mid \theta)$ for all data realization y.

- A property independent of data

Definition (Identification)

θ is identified if it has no observationally equivalent parameters.

- Define the equivalence mapping $K : \Theta \to 2^{\Theta}$, that is, $p(y \mid \theta) = p(y \mid \bar{\theta})$ for all y, if and only if $K(\theta) = K(\bar{\theta})$.

Definition (Reduced-form)

A continuously differentiable function $\phi(\theta) \in \mathbb{R}^n$ is called a reduced-form parameter if it is identified.

Example: Cochrane Model

Consider the full model

$$x_t = \rho x_{t-1} + \epsilon_t, \quad |\rho| < 1, \epsilon_t \sim N(0, \sigma_\epsilon)$$

$$i_t = r + E_t \pi_{t+1}$$

$$i_t = r + \psi \pi_t + x_t, \quad \psi > 1$$

Deep parameters are $\theta = (\rho, \psi, \sigma_\epsilon)$. The solution is equivalent to a AR(1) setting

$$\pi_t = \rho \pi_{t-1} + \frac{1}{\psi - \rho} \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

with reduced form parameters $\phi = (\rho, \frac{\sigma_\epsilon}{\psi - \rho})$, (ψ, σ_ϵ) not jointly identifiable.

The impulse response function is also not identified.

Theoretical Results

Brief

Under certain regularity conditions,

1. Theorem 1 (Sensitivity): The estimated set from the algorithm using a prior, characterizes the set of posterior means from the same class of priors, s.t.

   $\pi_\theta \in \Pi_\theta(\pi_\phi) := \{\pi_\theta(\phi(\theta) \in A) = \pi_\phi(A), \text{ for all } A \in \mathcal{A}\}$

2. Theorem 2 (Consistency): The estimated set from the algorithm, converge asymptotically to the 'true' identified set.

Discussion

In this paper, I attack the following problems:

- Estimation results of set-identified DSGE models are sensitive to choice of priors (Identification)
  - Pick any ‘reasonable’ prior, the algorithm is able to give results from other priors with the same predictive distribution
  - I also prove theoretically the validity of this method.
- Researchers are silent about non-identified DSGE models (Inference)
  - The complete identified set of parameters is given.
- Thank you!

Prior Sensitivity

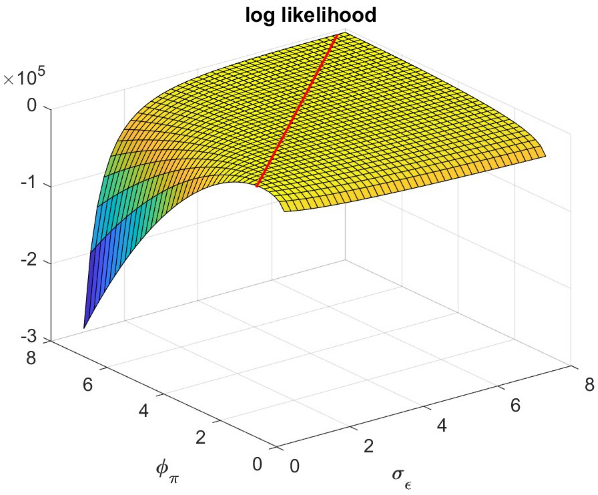


Figure: Likelihood function while fix $\rho = 0.8$

Prior Sensitivity

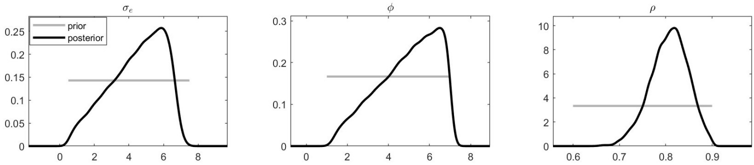


Figure: Cochrane model prior/posterior distribution with uniform priors

- The posterior of σ_ϵ and ψ are extremely informative even if only $\frac{\sigma_\epsilon}{\psi - 0.8}$ is identified.
- Reason? Joint likelihood density more concentrated on areas with higher values of ψ and σ_ϵ.
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